New AFL finals system is unfair!

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After much public criticism the Australian Football League (AFL) recently changed the system that it uses for the finals. The problem is that the new system it has adopted is still plagued with unfair arrangements of matches.

One of the main problems with the previous **McIntyre system** was that the arrangement of matches was unfair in the second round of the finals. After round one (r1) of the finals, the teams are reordered with the 4 winners from r1, preserving their previous relative order in the final-eight, followed by the 4 losing teams also in the same relative order as in the final-8. The two lowest ranked losers are eliminated from the competition, the two highest ranked winners have a bye in round two (r2), and the other teams play in r2, as illustrated in Figure 1 (T3vT5 and T4vT6), that is, the team now ranked third plays the team now ranked fifth, and the team now ranked fourth plays the team now ranked sixth.

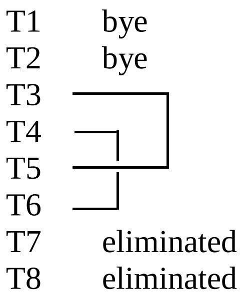

Figure 1: r2 matches in the previous McIntyre system

This arrangement of matches is unfair [1-3] because the team in third position (T3), which is ranked above the team in fourth position (T4), must play a better team (T5) than the team in fourth position, who plays T6. By the same token the fortunes of T5 and T6 are also inappropriately reversed. In the context of the 1999 finals, the 'Western Bulldogs' (T5) had to play 'Brisbane' (T3), whereas 'Carlton' (T6) had an easier opponent, in 'West Coast' (T4). Carlton was incidentally also advantaged by another rule, which states that, in every round, at least one match must be played at the MCG (Melbourne Cricket Ground).

In a deterministic system, that is a system where there is a specific rule about who plays who, it is important that when the teams are paired to play each other, that none of the lines which connect these teams, cross each other (as in Figure1), or otherwise a lower ranked team is necessarily given an easier task than a team ranked above it. This would be unfair. Our definition of fairness (in the first instance) means that there are no such matches, where a lower ranked team gets an easier match than a team ranked above it. If we were to strictly adhere to this principle this would only allow teams to be paired to play each other in an outer pairing arrangement (where the top team plays the bottom team, the second top team plays the second from the bottom team, and so on) in all rounds. Alternate arrangements, such as that shown in Figure 2, are also possible if any unfairness (that is, inappropriate benefit given to a lower ranked team in relation to a team ranked above it) is compensated by some other condition (such as, the higher ranked team cannot be eliminated). See discussion below in relation to Figure 2. It is also possible to have an unfair arrangement of matches in one round, compensated by another unfair of arrangement of matches in a subsequent round, but this is not practical here because two teams are elimінted in going from one round to the next, and a team that may have been unfairly treated may have been eliminated. Another interesting way to get around this problem is to randomly pair the teams to play each other [1-3]. These systems will be briefly discussed at the end of the paper.

The McIntyre System is also plagued with some other problems [1-3], such as the fact that the teams are not evenly matched in r1, where the top team plays the eighth team, and the second team plays the seventh team. These other tribulations are however a matter of choice, but this is not true when it comes to fairness. The rules of sport must be mathematically fair. The problem with the **new finals system** adopted by the AFL for the year 2000 is that it is also plagued with the same problem of unfairness as in the McIntyre system. This unfairness can now occur in both r2 and r3.

Under the new system, the 8 finalists are paired to play each other in r1 as shown in Figure 2. The top team plays the fourth team (A1vA4), the second team plays the third team (A2vA3), the fifth team plays the eighth team (A5vA8), and the sixth team plays the seventh team (A6vA7). These matches are respectively called the first qualifying final (QF1), the second qualifying final (QF2), the first elimination final (EF1), and the second elimination final (EF2). All matches are played at the home-ground of the higher ranked team. The top four teams cannot be eliminated in r1, whereas the two losers from the bottom half matches are eliminated.

It may seem unfair that the team in fifth position (A5) gets to play a home-final against a weaker opponent (A8), whereas the team ranked above it in fourth position (A4) must play the top team, away, but this is compensate by the fact that A4 cannot be eliminated in r1, and A4 remains above A5 even if A4 loses and A5 wins in r1.

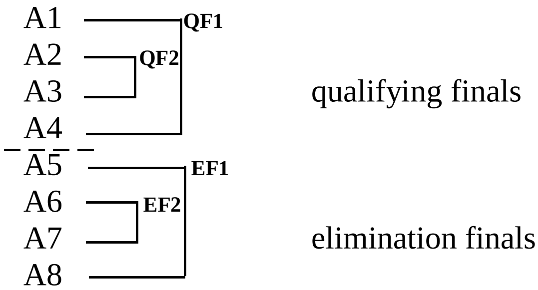

Figure 2: r1 matches in new finals system

In r2, the loser from QF1 (A1vA4) plays the winner from EF1 (A5vA8), and the loser from QF2 (A2vA3) plays the winner from EF2 (A6vA7), while the two winners from the qualifying finals have a bye. These two r2 matches are respectively called the first semi-final (SF1) and the second semi-final (SF2). In r3, the winner from QF1 (who had a bye in r2) plays the winner of SF2, and the winner from QF2 (who also had a bye in r2) plays the winner of SF1. These matches are called the first and second preliminary finals respectively. The winners of the two preliminary finals play in the Grand Final. The crossover used here, in that QF1 plays with SF2 and QF2 plays with SF1 (instead of QF1vSF1 and QF2vSF2) is deliberate so that the same teams do not play each other in r3 as played each other in r1. As we shall see below this arrangement of matches in r3 (as well as some of the matches in r2) can be unfair. It would seem that the AFL prefers to have an unfair arrangement of matches instead of having repeated matches in the finals.

Let us analyse the new finals system by listing all of the possible outcomes. The r1 outcomes can be enumerated by the notation (±,±,±,±), which respectively labels the outcomes to the 4 matches played in r1 (A1vA4, A2vA3, A5vA8, A6vA7). A "+" means the home team wins and a "–" means the away-team wins. (+,–,–,+), for example, means that the winners to the r1 matches were A1, A3, A8, and A6. The 16 possible outcomes to the r1 matches are listed below in Figure 3, where for each outcome, the six surviving teams are listed in order of merit, the two winners from the top half draw, followed by the two losers from the top half draw, then followed by the two winners from the bottom half draw, with the previous relative order in the final eight used to distinguish between teams of a similar status (that is the 2 winner of the QF's, the 2 losers of the QF's, and the 2 winners of the EF's). Figure 3 also shows how these teams are paired to play each other in the semi-finals under the new system.

One can see that the new system is unfair in r2 for each of the following r1 scenarios

(++–+), (–+++), (++––), (+––+)

(––+–), (+–––), (––++), (–++–)

[Or for (+*–*) and (–*+*), where "*" is a wild entry.] Consider for example the r1 scenario (–++–). Here (in r2) A1 must play a better opponent (A5) compared to A3, who is ranked below A1, but gets to play a weaker opponent (A7). Note that A3 did not win in r1 to earn this privilege. This scenario is similar to the Carlton scenario mentioned earlier in relation to the McIntyre system. Note that fairness is not restored if one swaps the two losers of the QF's with the two winners of the EF's and orders the two winners from the EF's ahead of the two losers from the QF's.

The new system can also be unfair in r3. For each of the r1 outcomes there are 4 possible outcomes to the r2 matches depending on who wins the two semi-finals. Consider for example the r1 scenario (++++) when all of the favorites win in r1. In this case A3vA6 and A4vA5 in r2, which is fair. We can label the two outcomes to the SF's by using the notation (±,±), where the first symbol tells us whether the home (+) or the away (–) team wins in the SF involving the highest ranked team (refer to Figure 3) and the second symbol refers likewise to the other semi-final.

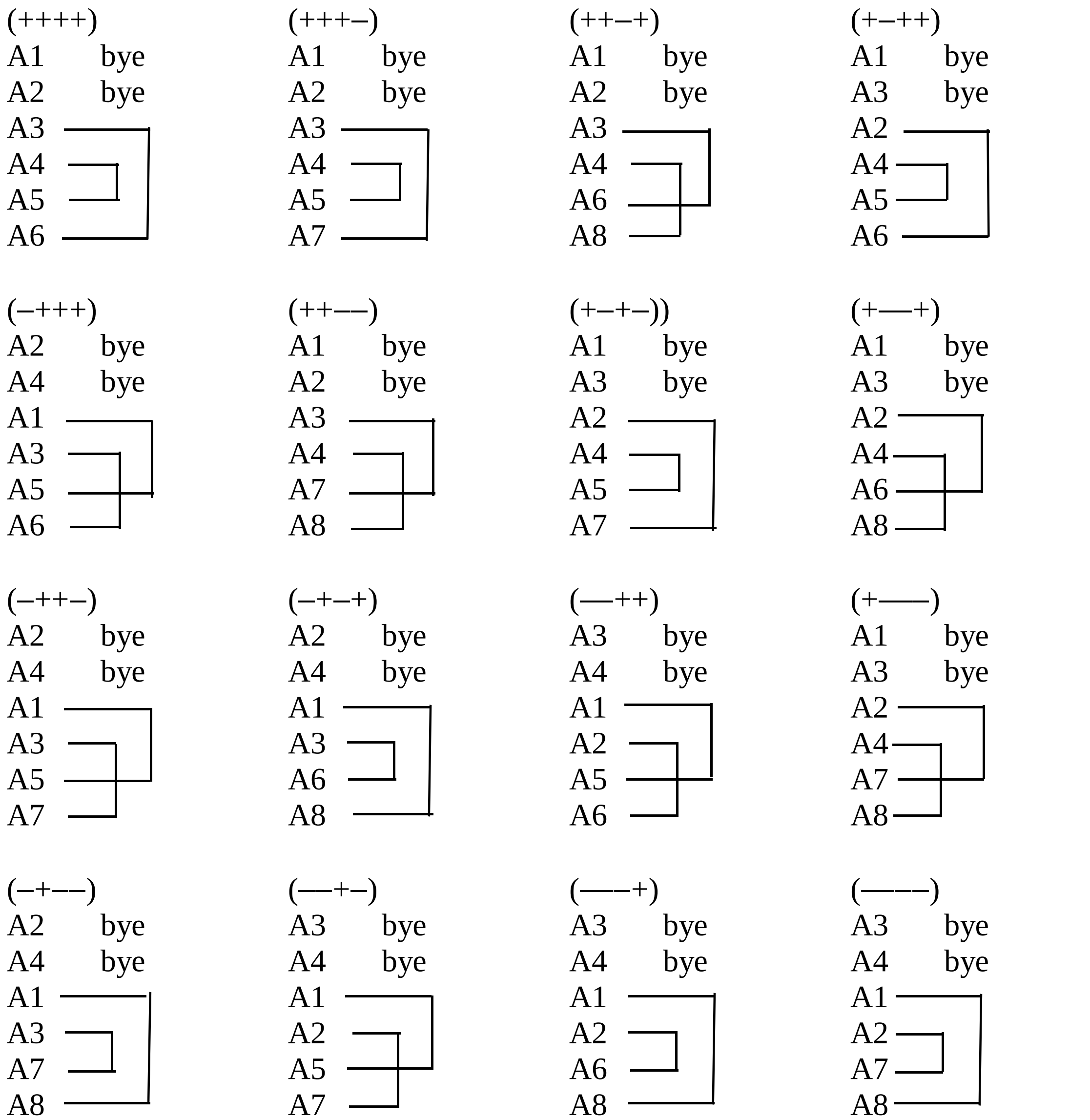

Figure 3: teams ranked in order of merit preceding r2 for all r1 outcomes (±,±,±,±), also showing r2 matches in new system

The 4 possible outcomes in the (++++) r1 scenario are listed in Figure 4, together with how these teams are paired to play each other in r3 under the new system. Note that the teams are listed in Figure 4 in order of current merit. One can see that in two of the listed outcomes, that is (++) and (+–), that the arrangement of matches is unfair. In the first case, A1 has to play a better team than A2, even though A1 is ranked above A2. It would seem to be preferable to finish in second position in the final-8 to gain this advantage in r3, in this case. Note that A1 did not lose its r1 match to be disadvantaged in this way.

It turns out that for all of the other r1 outcomes that are perceived to be fair (see Figure 2), that in ½ of the possible outcomes to the r2 matches, the arrangement of matches in r3 is unfair.

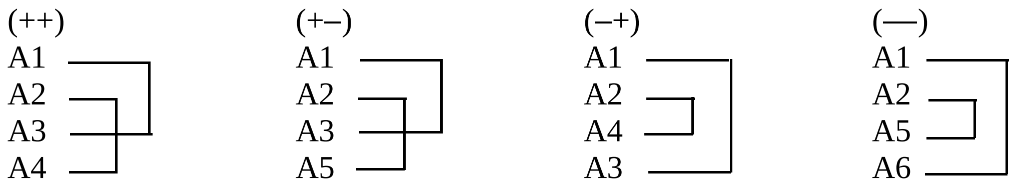

Figure 4: possible r2 outcomes to (++++) r1 scenario also showing r3 matches in new system

For the r1 outcomes that are unfair in r2, it turns out that the r3 matches in the new system are also unfair in 3 out of the 4 possible outcomes to the r2 matches. Consider for example the r1 outcome (+—+), where the r1 winners are A1, A3, A6 and A8. The teams are (unfairly) paired to play each other in r2 as shown in Figure 2, that is A2vA6 and A4vA8. [Why is A4 advantaged over A2?] The order of merit preceding r3 and the way that the teams are paired to play each other in r3 are shown in Figure 5 for the 4 possible outcomes to the r2 matches. The absurdity of the situation is highlighted by the outcome (+–), where A1 plays A2, whereas A3 plays A8. Why is A3 advantaged over A1 in this way? A1 did not lose its r1 match. In the correct (that is fair) arrangement of matches in this example, A1 should play A8 and A3 should play A2. The fundamental reason why the AFL has swapped the way that the teams should play each other in r3 is so that that the same teams do not play each other again in r3 that played each other in r1.

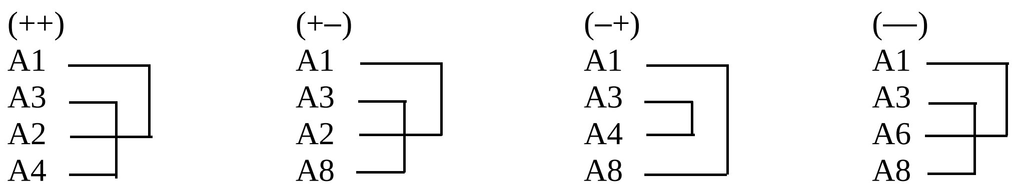

Figure 5: possible r2 outcomes to (+—+) r1 scenario, also showing r3 matches in new system

One should also note that the teams that are treated inappropriately in r2 are not compensated in r3 by a reversal of fortunes in the unfair scheduling of matches. In the example depicted in Figure 5, that is the (+––+) scenario, A2 was treated unfairly in relation to A4 (see Figure 3), and is treated unfairly again in r3 in relation to A4, when both teams win (++). In scenario (+–) A2 even cops it in relation to A8. In this r1 scenario A6 is also inappropriately disadvantaged with respect to A8 in r2, and is further disadvantaged with respect to A8 in r3 for the r2 outcome (––).

Figure 6 lists the number of times each team is inappropriately disadvantaged or advantaged in r2. In the r1 outcomes where A1 is inappropriately disadvantaged in r2 matches [that is (–+++), (–++–), (––++), and (––+–)], it is also inappropriately disadvantaged in r3 if it wins. In the games where A1's position is neutral [that is, when it wins, corresponding to outcomes (+***), *=wild] A1 is inappropriately disadvantaged in r3 with probability ½ and ¾ respectively, depending on whether there was an fair or unfair arrangement of matches in r2. A4 on the other hand, who is inappropriately advantaged in r2 4 times, is also inappropriately advantaged in r3 matches with probability ½ for all outcomes except for the outcomes (–+++), (–++–), (––++), and (––+–) where it is inappropriately advantaged with probability ¾.

Further to the above, if A1 and A4 are both Victorian teams (or teams from the same state), there is no home-ground advantage in r1, and there is absolutely no difference between finishing first or fourth on the ladder. The probability that this occurs is 47/120 ≈ 0.39. This problem may also occur with any of the other r1 pairs (A2,A3), (A5,A8) and (A6,A7), and also in the r2 and r3 matches. The probability that such a match will occur in the finals is 0.97

	number of times inappropriately advantaged in r2	number of times inappropriately disadvantaged in r2	number of times situation neutral
A1	0	4	12
A2	2	2	12
A3	2	2	12
A4	4	0	12
A5	0	4	4
A6	2	2	4
A7	2	2	4
A8	4	0	4

Figure 6: some statistics for r2 (new system)

As noted above the AFL seems to have given undue precedence to the avoidance of rematches in the finals, at the expense of fairness. An unfair match is expected to occur 3 times in every 4 years, with unfair matches occurring in both r2 and r3 3 times in every 8 years. We believe that a fairer of arrangement of matches is certainly preferable (if not essential) to having a rematch in the finals. In the final-4 system used in the West Australian football competition (Westar Rules), and in the final-five system used in the VFL (Victorian Football League, the competition preceding the AFL), rematches are or where common place.

A detailed analysis of ‘fair’ finals systems, with no byes (so the public also gets to see the best teams play in r2 and there are 10 finals instead of 9) has been undertaken in references [1-3]. In these systems, the teams are reordered after each round and the two lowest ranked losers are eliminated. With deterministic rules there are basically only two possible systems allowed that are fair [1-3]: the ‘**outer pairing system**’, where the outer pairing principle is used in each round, and the ‘**split pairing system**’, where the teams are paired to play each other in r1 as in Figure 2, followed by outer pairing in r2 and r3. It is not possible to have a fair system without rematches. Both systems may have r1 rematches in r3. In the ‘outer pairing system’, this occurs with probability 1/8, and in the ‘split pairing system’ with probability ¼. Fair systems with byes in r2, on the other hand will have a rematch occurring with either probability ½ or 1.

Another interesting possibility is to consider stochastic systems [1-3], where the teams are randomly paired to play each other, possibly under certain conditions, for example, a team from the top half might be required to play a team from the bottom half. In the most general of such possibilities, the teams may be randomly paired to play each other from any ladder position in all rounds. Once again the two lowest ranked losers are eliminated in going from one round to the next. The fairness criteria is avoided in these systems by the introduction of an element of chance, or the luck of the draw. It is important to note that these systems give higher ranked teams a relatively higher probability to play a weaker team and to host a home-final. The higher ranked teams also have a relatively higher probability to proceed to the next round, because they are more likely to be the highest ranked losers when they do lose. The premiership probabilities in this ‘**general random pairing system**’, where the teams are randomly paired to play each other from any ladder position in all rounds, are given in Figure 7. They are uniformly graded from top to seventh/eighth position. Rematches can also be avoided in this system if desired [1], and other conditions can also be imposed, such as the top two teams should not play each other in rounds 1 and 2.

	McIntyre	new AFL	outer	split	random
A1	18.75%	18.75%	18.75%	20.31%	18.75%
A2	18.75%	18.75	17.19%	18.75%	17.68%
A3	15.625%	18.75	14.45%	18.75%	14.91%
A4	12.5%	18.75	12.11%	17.19	13.28%
A5	12.5%	6.25%	12.11%	6.25%	11.00%
A6	9.375%	6.25%	9.77%	6.25%	9.38%
A7	6.25%	6.25%	7.81%	6.25%	7.50%
A8	6.25%	6.25%	7.81%	6.25%	7.50%

Figure 7: premiership probabilities in various systems considered.

Some of the criteria that can be used to judge finals systems are: fairness (essential in our view); no rematches (especially in consecutive rounds): close games (teams chosen to play are well matched); realistic premiership probabilities for all finalists (particularly when home-ground advantage and form are taken into account [1]); and graded premiership probabilities that reward teams for finishing higher up the ladder. The best systems that satisfy these criteria are the split pairing and general random pairing systems.

References

[1] Christos, G.A., ‘More competitive finals systems for the AFL and other team sport competitions’, Curtin University Technical Report 22/97, November 1997.

[2] Christos, G.A., ‘The AFL finals: It’s more than a game’, proceedings of the Fourth conference on Mathematics and Computers in Sport, Bond University, July 1998, Ed. N. de Mestre and K Kumar, pp 111-124.

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