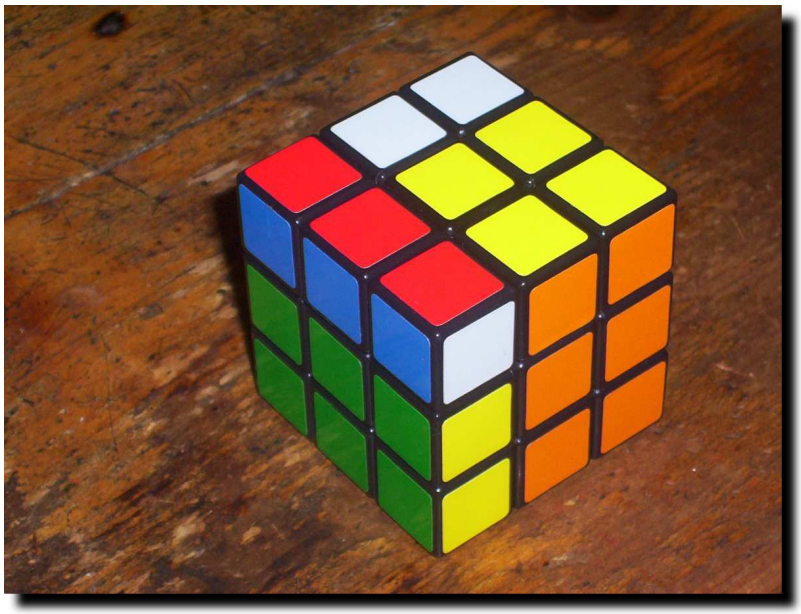
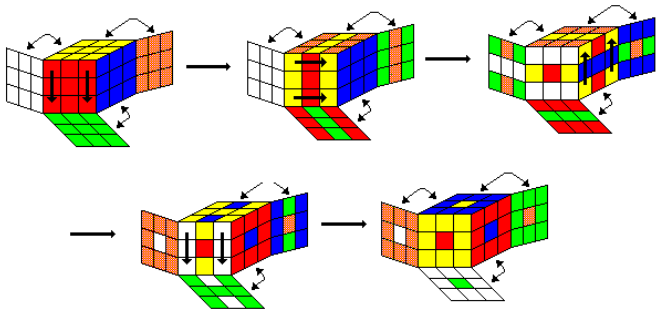
NEW solution to Rubik's cube

Dr George Christos



fully illustrated, step by step



Introduction

Rubik's cube was a sensation when it was released in the late 1970s. Many people were however unable to solve the puzzle, even with the aid of published solutions. Most of these solutions were generally too technical around the end of the solution. This book offers an alternate/different solution to Rubik's cube, devised by the author in 1980, whilst a DPhil student at Oxford University, but has never been previously published. The book contains some 36 detailed colour illustrations, which we hope will enable those, who were previously unsuccessful, to finally complete the puzzle. With a little practice, you should be able to do the puzzle in about 1 to 3 minutes with ease. I have personally gone under a minute a few times.

Previous solutions start by doing one face and then proceed by layers from there, working their way up the cube. In this booklet, we do opposite faces simultaneously. This gives you much more freedom in completing the last part of the puzzle, unlike other solutions which become exceedingly more complicated around the end.

Basics

The cube has six sides, each of which has 9 small coloured faces. Upon dismantling the cube (see below how to dismantle the cube), it is apparent that the cube is built around a central structure with six protruding arms which connect to the six coloured centre faces of the six sides of the cube. The cube has 8 corner pieces, each of which have three small coloured faces, and 12 edge pieces, each of which have two small coloured faces. See illustration in Figure 1. These pieces of the cube are held together by the relative pressure of each piece against the others. The mechanics of Rubik's cube is as fascinating as the puzzle itself. Figure 1 also shows how the different slices of the cube can be rotated. These rotations, also referred to as moves, mix corners with corners, and edges with edges, but corners cannot be transformed into edges, or vice versa. This mixing of corner pieces and edge pieces is so profound that it only takes about 7 or 8 moves away from the unscrambled position to completely scramble the cube beyond recognition. The reader should also note that two rotations carried out in the reverse order do not necessarily have the same effect on the cube. Have a play with the cube, and don't be afraid to lose yourself as you can always dismantle the cube and put it in the finished position.

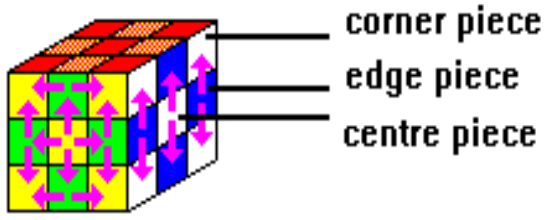


Figure 1

The cube can be easily pulled apart by turning one face by 45 degrees (or half of a rotation) and lifting out one of the edge pieces of the rotated face. The rest of the cube should then come apart quite easily. When reassembling the cube ensure that you turn the last face at a 45 degree angle while you fit the last edge piece back in. Dismantling and reassembling the cube in the unscrambled position is particularly useful as you err while developing and studying solutions to the cube. This is how I developed the sequence of algorithms to solve the puzzle in 1980.

The main idea of the puzzle is to restore the cube from a scrambled position as in Figure 2 to the unscrambled state where all of the same coloured little faces are on the same side of the cube. The centre faces of each side of the cube are relatively fixed and this gives an indication of where one needs to move the other similarly coloured small faces to complete the puzzle. A solution to a complicated puzzle such as Rubik's cube requires a systematic approach. Previously published solutions generally proceed by getting one face of the cube correct, say the bottom face, and then aim to slowly develop the solution by working layer by layer up to, and including, the top face. The problem with these solutions is that the last few steps in the solution are generally quite complicated since one is trying to preserve all of the 'hard work' that has already been done. In our solution we aim to get the corner pieces correct (or approximately correct, as will become clear below) followed by the edge pieces. This leaves a considerable amount of freedom almost right up to the final solution of the puzzle.

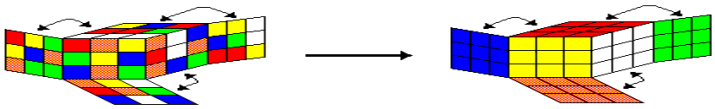


Figure 2

Basis of our solution

In the process of our solution we aim to initially restore the cube to a situation as shown in Figure 3, where opposite colours on the cube are allowed to mix. For the cube shown we will attempt to get all of the red and orange small faces on two opposite sides of the cube, ditto for the blue and white faces, and the yellow and green faces. Once the cube has been placed in this state it is relatively easy to complete the solution. Most cubes, when they first came out, had red and orange faces (or centres) opposite to each other, and with green and blue opposite, and white and yellow opposite. There are however now other variations, in what colours are opposite to each other, and some have different colours. If you have a cube with different colours opposite each other, you will need to make the necessary adjustments to the discussion that follows.

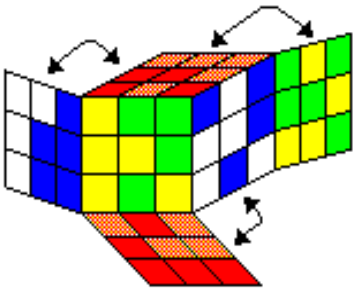


Figure 3

Restoring a pair of opposite faces

In the following we will develop a solution to Rubik's cube puzzle from a completely scrambled situation. Our first task will be to get two opposite sides of the cube to only consist of the two opposing colours, corresponding of course to the two colours on the small centre faces. If we consider, for example, the red and orange colours, we aim to get the cube into the configuration shown in Figure 4. To achieve this we need to first get all of the corresponding corner pieces into the 'correct' position. In keeping with our theme, outlined previously, we will not distinguish between red and orange, and so in the following we will represent these two colours by 'red' alone. Later on, when we get the cube into the position shown in Figure 3 we will relinquish this imposed colour blindness, and try to get all the red faces onto on side and all the orange faces onto the other opposite side. In the figures that follow, if any faces are not coloured then they are not relevant to the current discussion. The method developed here can also be applied to any of the other two opposing pairs of colours to begin with. There is no reason why we have to start with the red and orange coloured faces.

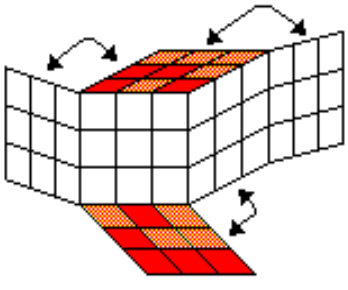


Figure 4

An important algorithm (or sequence of moves) that enables one to get all of the 'red' corner faces into the 'correct' position is shown in Figure 5. This trick (called Trick #1) essentially rotates two 'red' corner faces (or sometimes one, as may be sometimes required; see discussion below) so that the 'red' corner faces are on the sides of the cube with the red and orange centres, without losing any of the 'red' corner faces that are already correct. A double

move/rotation is represented in these diagrams by a line with a double arrowhead.

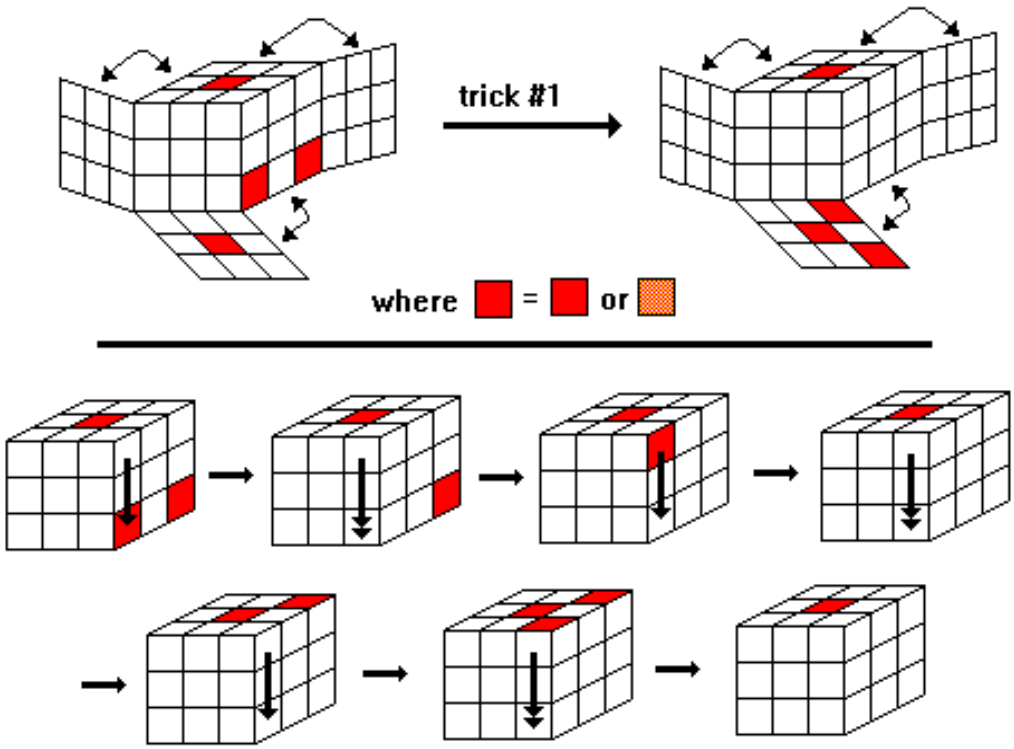


Figure 5

Let us begin. Look at the cube, and move any obvious red or orange corner faces onto the correct sides of the cube, with the orange and red centres. At this stage do not worry about the 'red' edge faces. You should be able to get at least 4 or 5 of the 8 corner faces on the correct sides without too much trouble. Be careful when you are rotating the cube slices to gain more 'red' faces that you don't lose more 'red' corner faces than what you are gaining.

If you hold the cube so that a 'red' centre points upwards then you can rotate any vertical slice of the cube by a double rotation, and any horizontal slice by a single rotation without losing any of the 'red' corners that you have already placed in the correct position. This 'freedom', illustrated in Figure 6, is sufficient to enable you to place the cube in the starting position to use Trick #1, so that you can restore all of the other 'red' corner faces. We illustrate this technique with two examples in Figures 7.

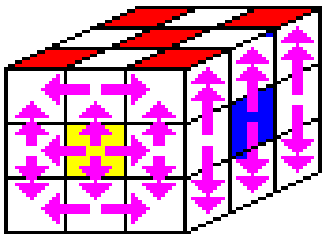


Figure 6

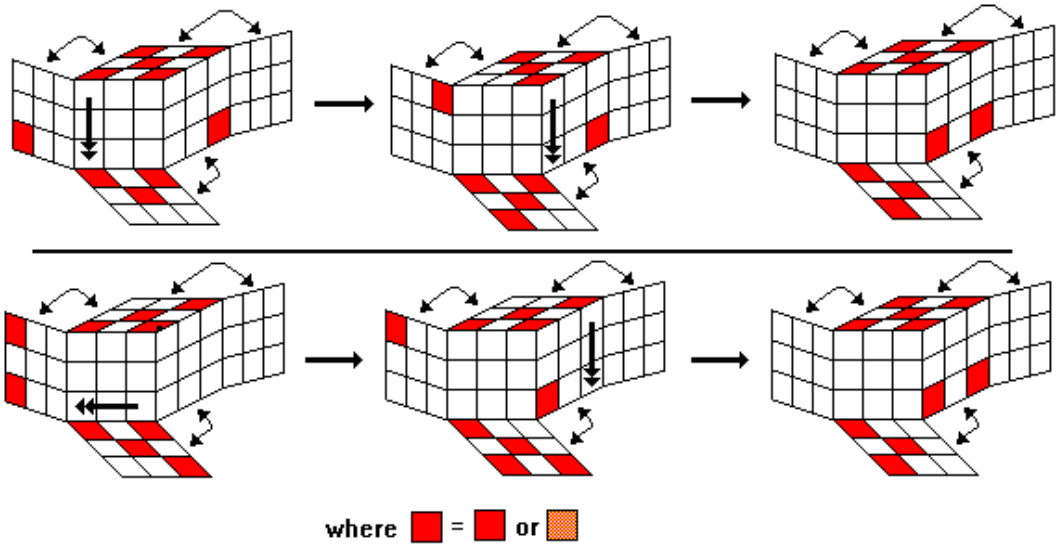


Figure 7

Trick #1 is also useful in the situation shown in Figure 8 where only one corner faces needs to be restored. Note that the other faces marked with a 'cross' are neither red nor orange. This situation frequently arises when you have 3 or 5 correct 'red' corner faces. You will find that it is never possible to have 7 red and orange corner faces correct. The last step in getting all of the red and orange corner faces correct will always involve two faces. It is therefore necessary in these cases to restore at some point only one 'red' corner face.

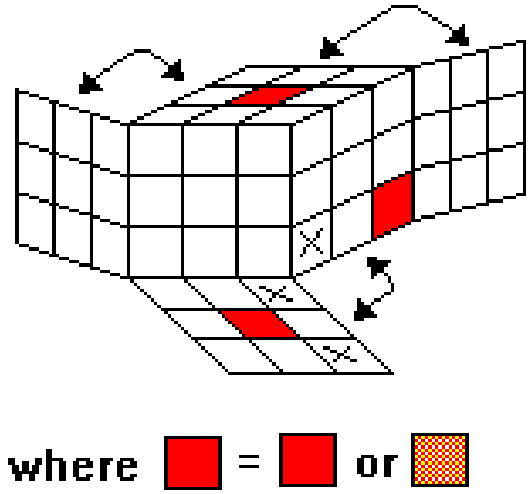


Figure 8

The next step in the solution of the cube is to get the 'red' (red and orange) edge faces in the correct position. Tricks #2 and #3, shown in Figures 9 and 10 respectively, are particularly useful for this purpose.

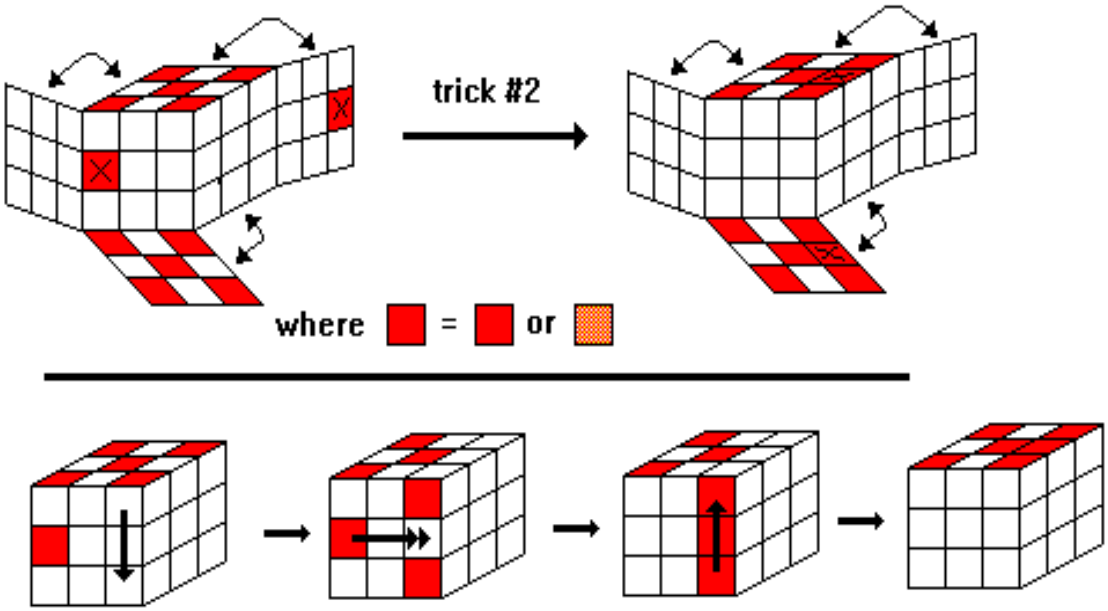


Figure 9

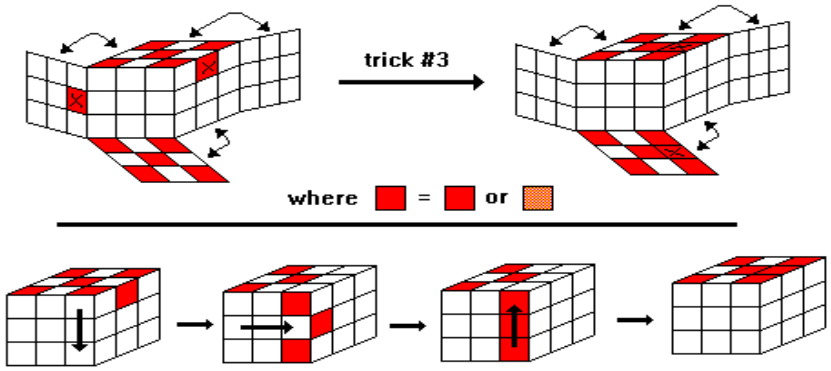


Figure 10

You may need to use the available 'freedom' shown in Figure 6, to place the cube in a starting position so that you can apply Trick #2 or Trick #3. The new feature here is that the last step may require the restoration of only one 'red' edge face. This 'red' face can be located anywhere marked with a "cross" or a "naught" as shown in Figure 11. If the missing 'red' face is in the position marked with a "naught" then the Trick # 4, shown in Figure 12, will be useful. If the missing 'red' face is in the position marked with a "cross" then Trick # 5, shown in Figure 13, will be useful. The idea behind Tricks #4 and #5 is that you essentially lose one more edge face and then apply Trick #2 or Trick #3. You should now have the cube looking as in Figure 4 with all of the red and orange faces on either the red or the orange sides of the cube.

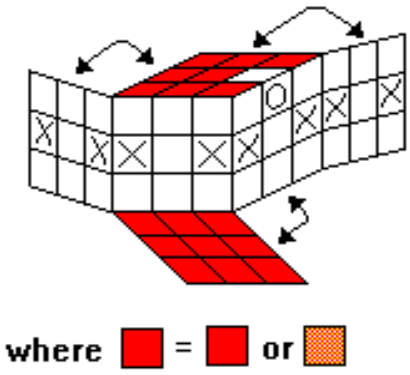


Figure 11

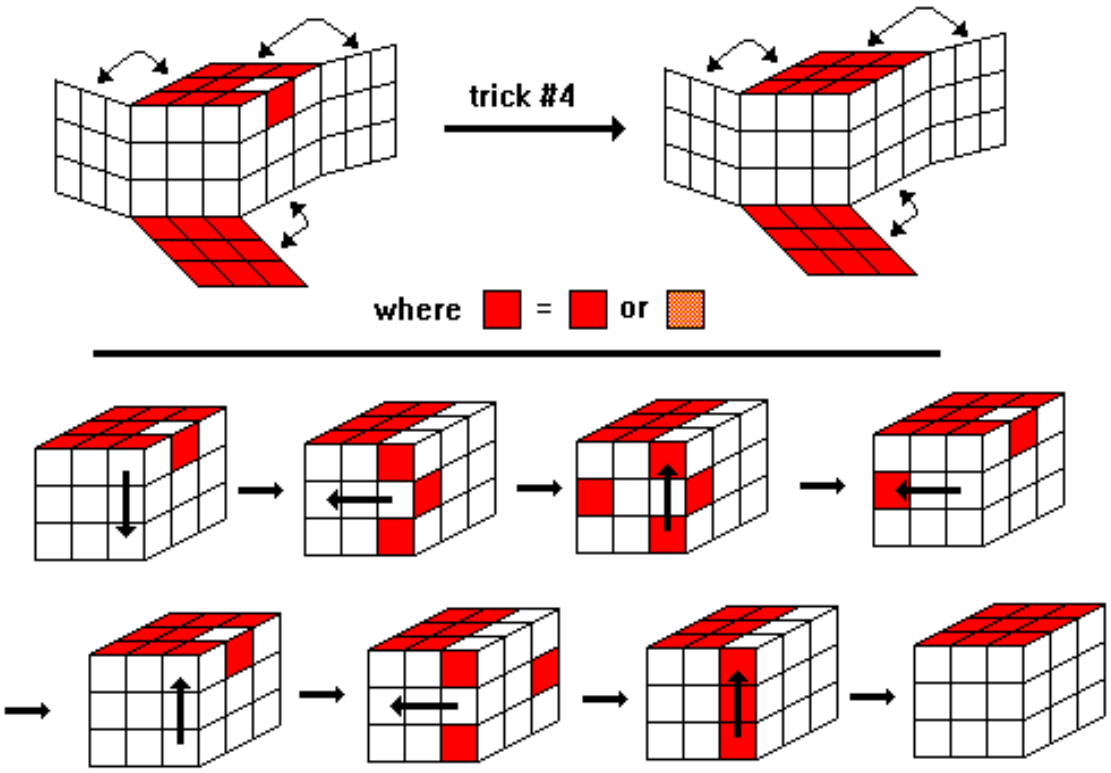


Figure 12

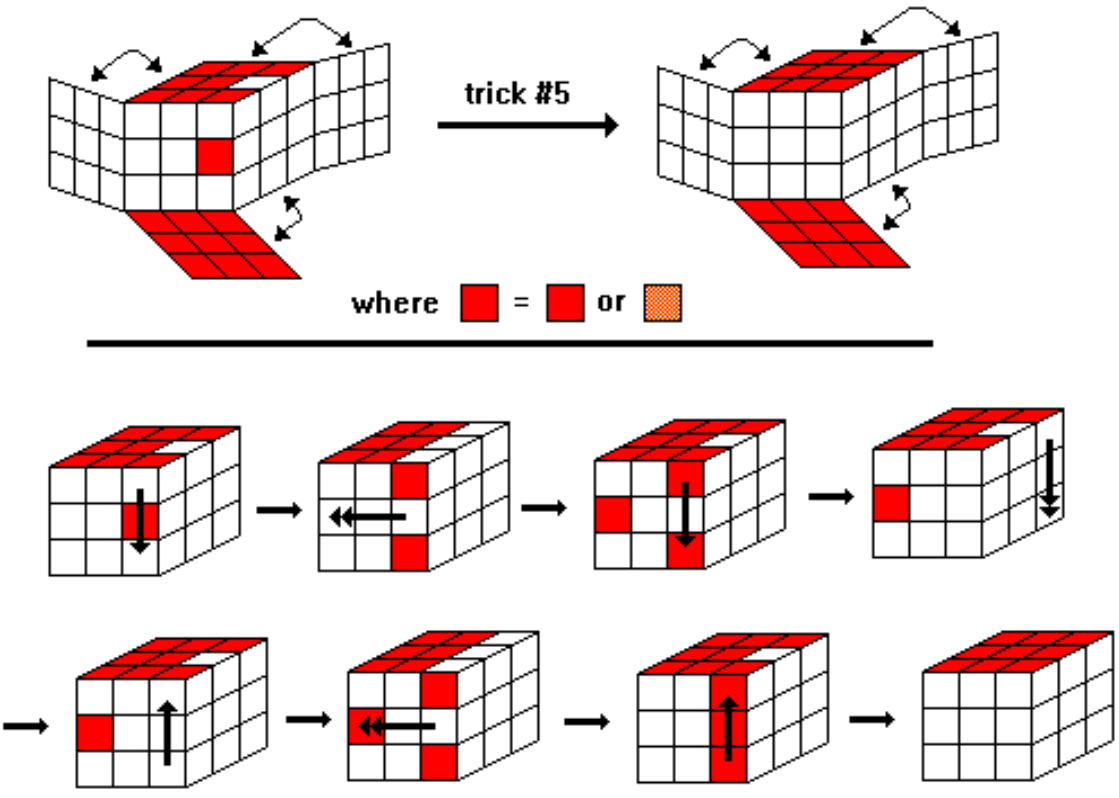


Figure 13

Restoring the other opposite pairs of coloured faces

We now proceed to restore another pair of colours on opposite sides of the cube. Let us consider the blue and white coloured faces, which we will collectively refer to as the 'blue' faces. To begin with we must get the 'blue' corner faces into the correct position. Use the 'freedom' moves shown in Figure 6 to get as many 'blue' corner faces as possible in the correct position. The remaining 'blue' corner faces can be restored by using Trick #6, shown in Figure 14. This is in essence just a variation of Trick #1. The cube can be placed in a position where this trick can be used by using the new set of available 'freedom' moves, which are now a bit more restricted. At this stage, a double rotation of any slice of the cube will not lose any of the 'red' faces, or any 'blue' corner faces that you may already have correct. Figure 15 illustrates how to use this freedom to prepare the cube for the application of Trick #6.

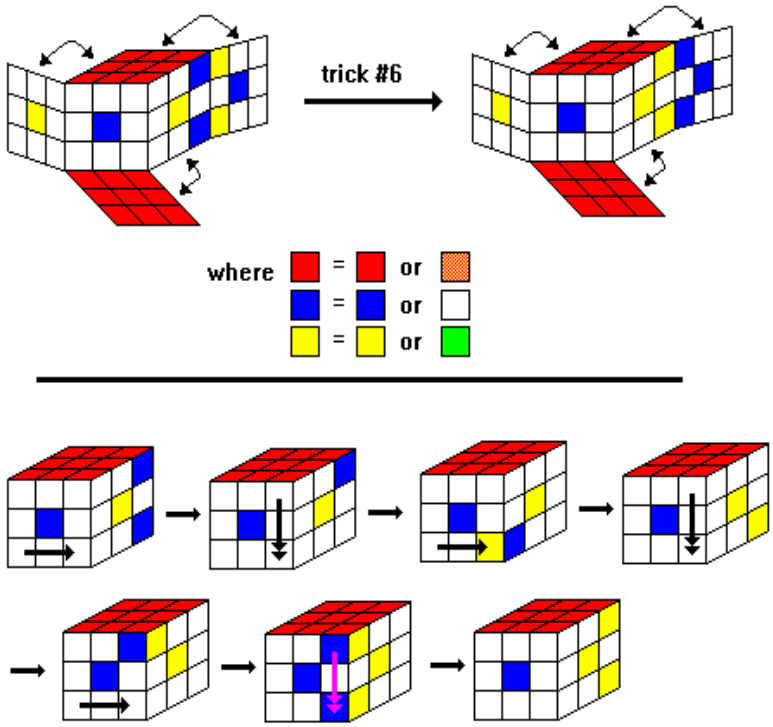


Figure 14

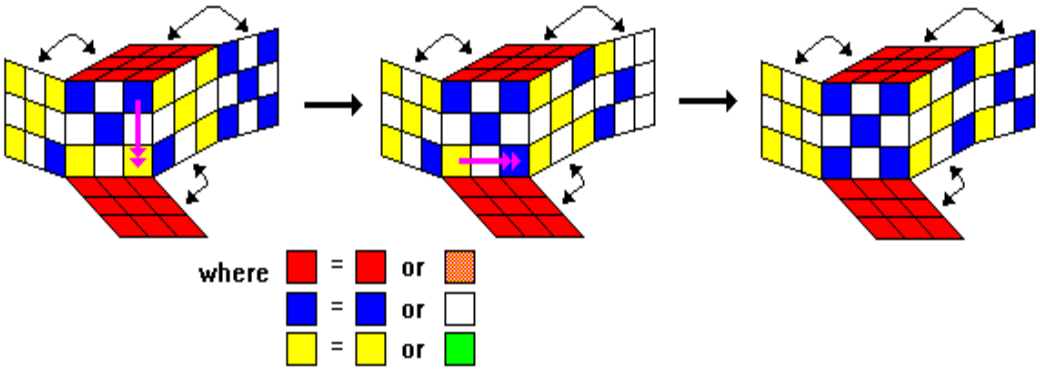


Figure 15

Your cube should now look like that in Figure 16 where all of the 'red' faces are correct, and all of the 'blue' corner faces are correct. You will notice that

the yellow and green corner faces are automatically correct. The next stage of the solution requires you to get the 'blue' edge faces on the correct sides of the cube. We will distinguish here between the edge pieces that are connected to a small red or orange face and those that are not. The 'blue' edge faces that are connected to a small 'red' face, as marked in Figure 16, can be restored by using Trick #7, shown in Figure 17. Once again the freedom that is available to you at this time (any double rotation) can be used to get the cube into position to use Trick #7. You may need to apply this trick twice since there are four such edge faces and the trick only restores two edges at a time.

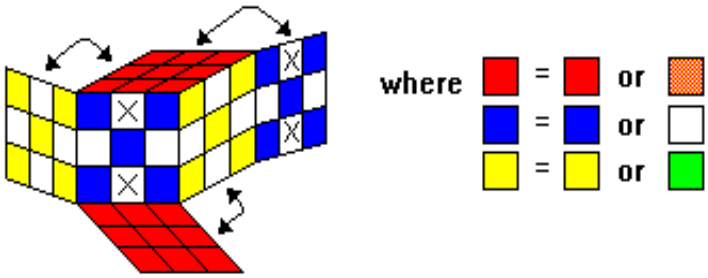


Figure 16

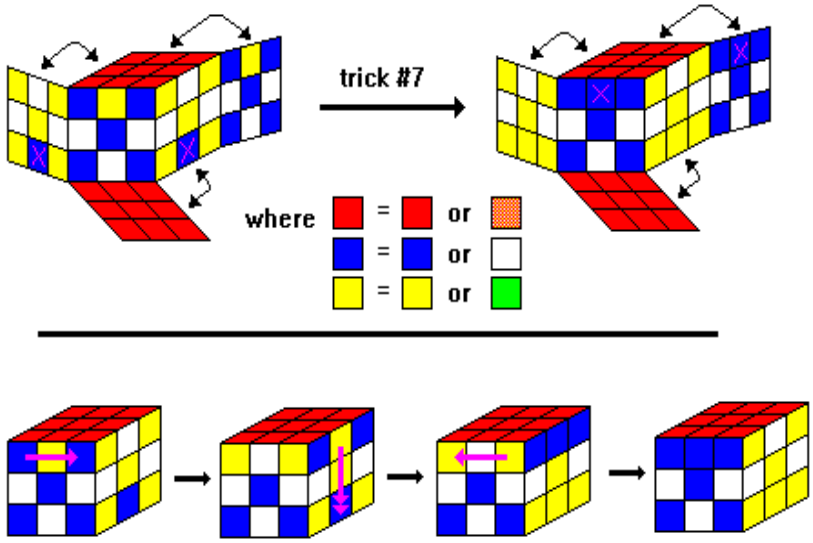


Figure 17

As before you will sometimes encounter the situation where there is only one blue or white edge face connecting to a red face that is not correct. In this case you need to essentially lose one more edge face so that the last two edge faces can be restored simultaneously. Trick #8, shown in Figure 18 illustrates how to do this.

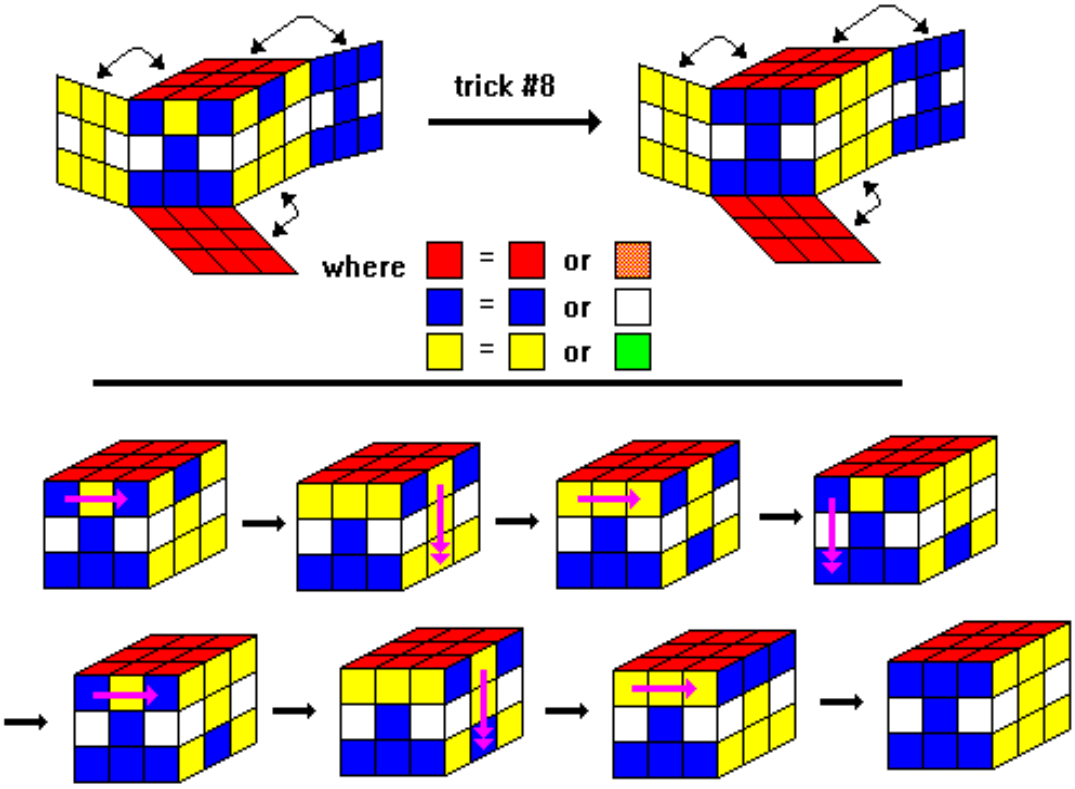


Figure 18

It now remains to restore the 'blue' edge faces that are not connected to a red face. These faces can be located in the positions marked in Figure 19. Trick #9, shown in Figure 20, will restore two such 'blue' edge faces to their correct positions. You may need to use this trick twice, since there are four such pieces to correct. Once again, to use this trick you may need to reposition the incorrect pieces using the available freedom moves.

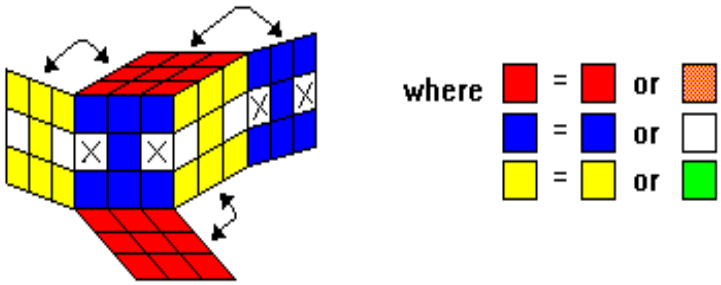


Figure 19

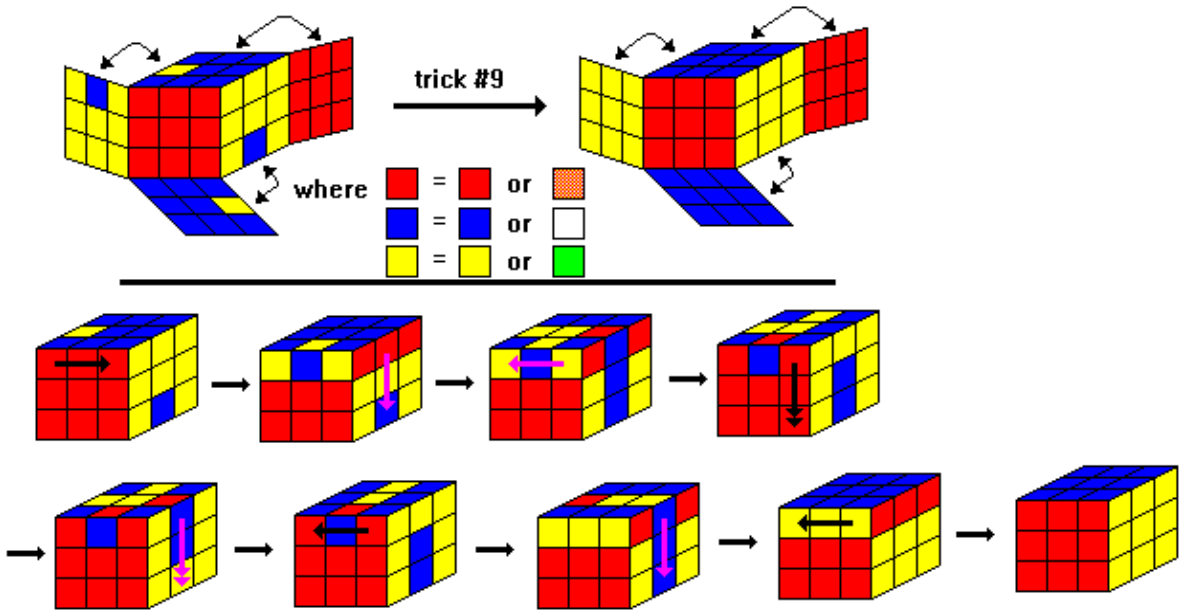


Figure 20

You should now have the cube in the configuration shown in Figure 3, where two opposite faces of the cube are entirely comprised of red and orange small faces, another two opposite faces of the cube are comprised of blue and white small faces and the other two opposite faces of green and yellow small faces.

The twisted cube

From now on we will relinquish our imposed colour blindness that did not distinguish between red and orange, between blue and white, and between green and yellow.

The next step in the solution requires the identification of whether there is a 'twist' in the cube. If you look at the red and orange faces you can use the 'freedom' double rotations to see if you can get all of the red corner faces on the red side of the cube. Try it. If you can do this then the red-orange direction does not have a twist associated with it, otherwise we say that the red-orange direction has a 'twist'. Now apply this method to the other pairs of colours, that is, the blue-white pair and the yellow-green pair.

There is a very easy way to determine if there is a twist in any of the three directions of the cube. Count the number of red corner faces on either the red or the orange sides of the cube. If there is an odd number of small red or orange corner faces on either face, then that direction has a twist. If however there are two red corner faces on the red and orange cube faces then there may or may not be a twist in that direction. Use double rotations to get two red corner faces adjacent to each other. If the situation is that the two red corner faces at each end are both vertical or both horizontal, as shown in Figure 21 then there is no twist. If however one pair is vertical and the other horizontal, as shown in Figure 22, then there is a twist in that direction. This notion can also be applied to the other directions, that is the blue-white direction and the green-yellow direction. We illustrate these

notions with some examples in Figure 23. You should now determine how many twisted directions there are in your cube. You will find that there are, either, no twists, two twists or three twists. It is impossible to have only one twisted direction. We will now develop techniques to eliminate the twists in your cube. If your cube is not twisted then you can skip the next section (Untwisting your cube) and proceed to the final step in the solution.

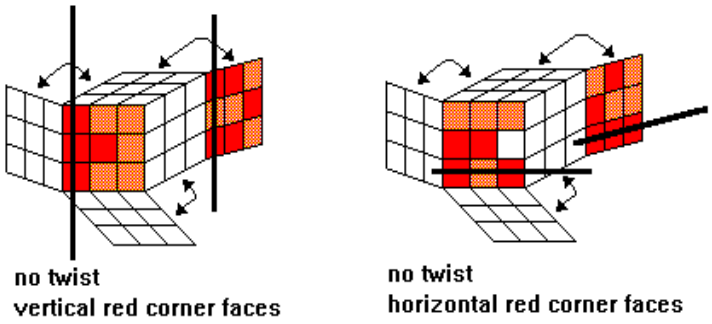


Figure 21

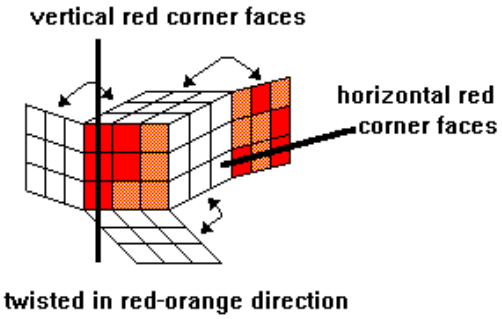


Figure 22

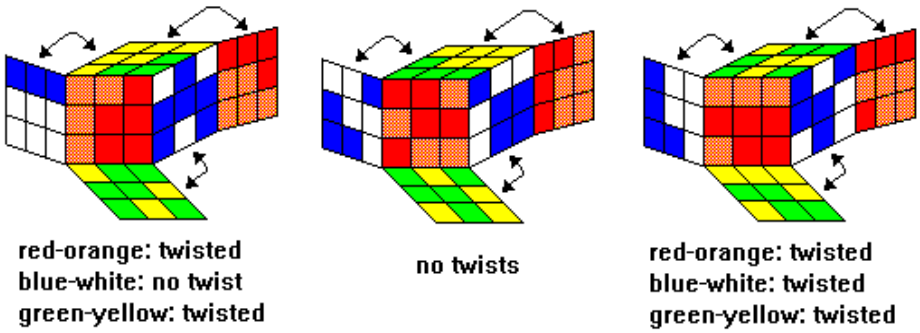


Figure 23

Untwisting your cube

If your cube has three twisted directions then there are generally two such possibilities, but we do not really need to distinguish between them. The twists in the cube are eliminated by using Trick #10, shown in Figure 24. This can be applied to any orientation of the cube, which explains the absence of any colours in Figure 24. You essentially take the cube in any orientation and perform Trick #1 to swap over two corner faces from one set with another.

You then rotate the whole cube in your hand, and repeat the above to restore the corners that were deliberately lost. Now look at your cube. The twists are either still there or they have been removed. If the twists are still there then you will need to repeat Trick #10, with the cube in any orientation. The twists in your cube should now have been removed.

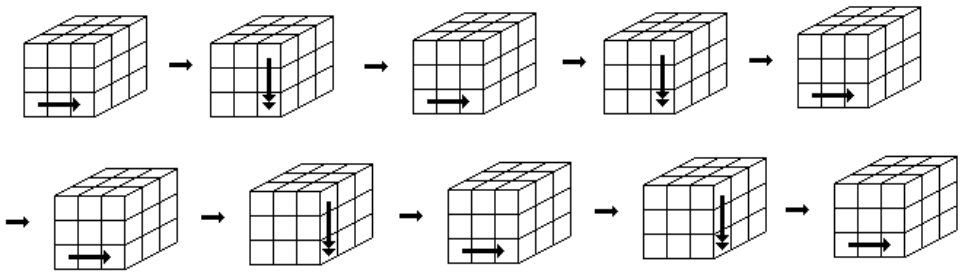


Figure 24 trick #10: untwisting a cube with three twists

The last situation that we need to consider is <u>when the cube has a twist in two directions only</u>. To undo these twists you need to pick any one of the twisted directions and then follow the procedure shown in Figure 25 to attempt to untwist one of these directions. This leaves two corner face pairs (that is four in all) swapped and these must now be restored by Trick #11, shown in Figure 26. If you place your thumb on the purple coloured square shown in this figure you can complete this sequence of moves without repositioning the cube in your hand. After a while you will be able to do this with your eyes closed. [Note that Trick #1 will not work in this case since it will retwist the cube.] This trick will need to be used twice, as shown in Figure 27, for the corresponding example in Figure 25. One edge face is always left incorrect and you will need to use Trick #8 to restore this.

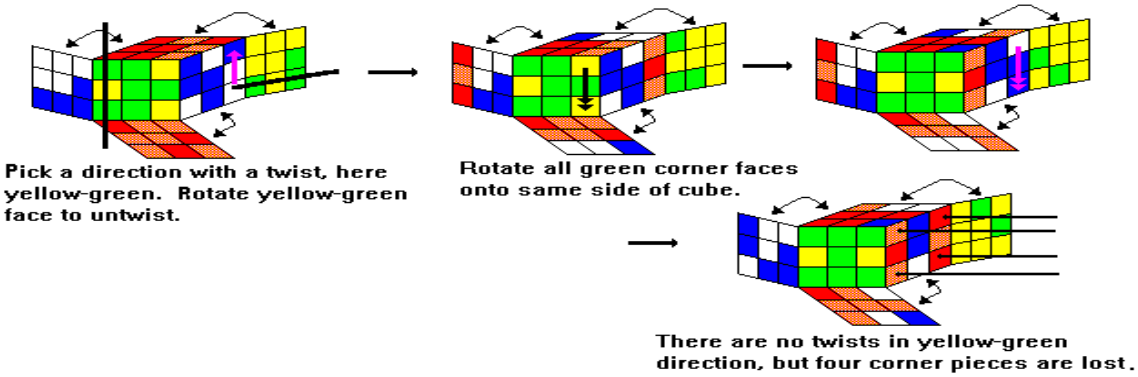


Figure 25

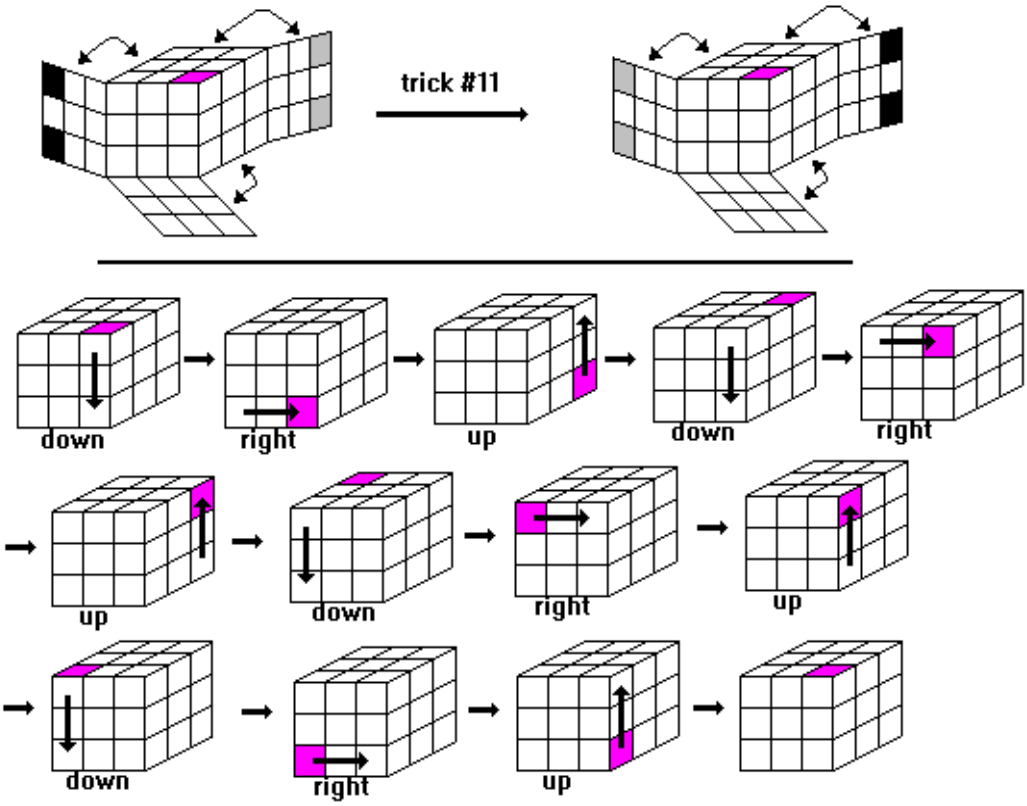


Figure 26

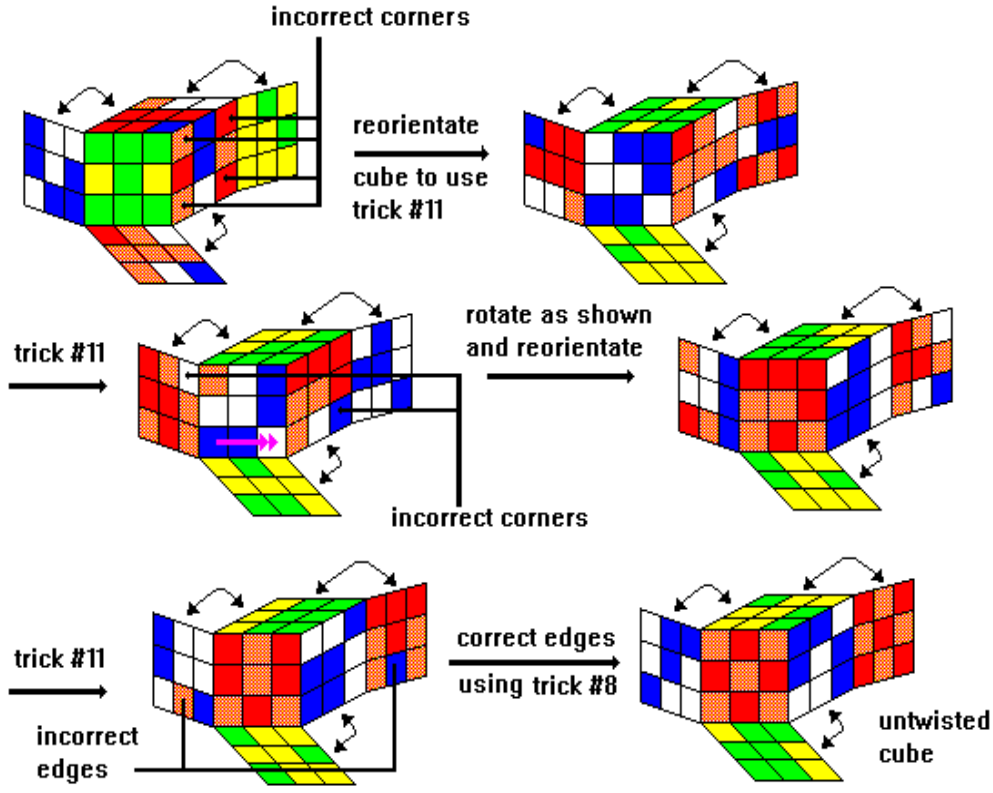


Figure 27

The final step

The last part of the solution is to restore the cube from the situation where only the opposite colours of the cube, such as red and orange, etc. are mixed

with each other only, and there are no twists in the cube as defined above. Note that you can freely rotate any slice of the cube by a double rotation without mixing anything other than red with orange, blue with white, and yellow with green.

The first step in this part of the solution is to pick on any coloured face, and then to restore all of those coloured faces to that side of the cube which has the corresponding coloured centre face. Ideally you should choose the colour that has most of its faces already in the correct position. For our discussion we will suppose that the red face is the most appropriate. We want to get all of the red faces onto the side of the cube which contains the red centre face. Begin with the red corner faces. You will either have them all correct, 2 correct, or none correct. If none are correct then double rotate any two parallel side faces as shown in Figure 28. If two red corner faces are correct then the other red corner faces can be restored by double rotations. All of the possible cases that may arise are covered in Figure 29.

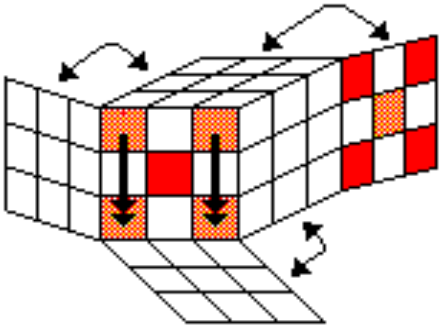


Figure 28

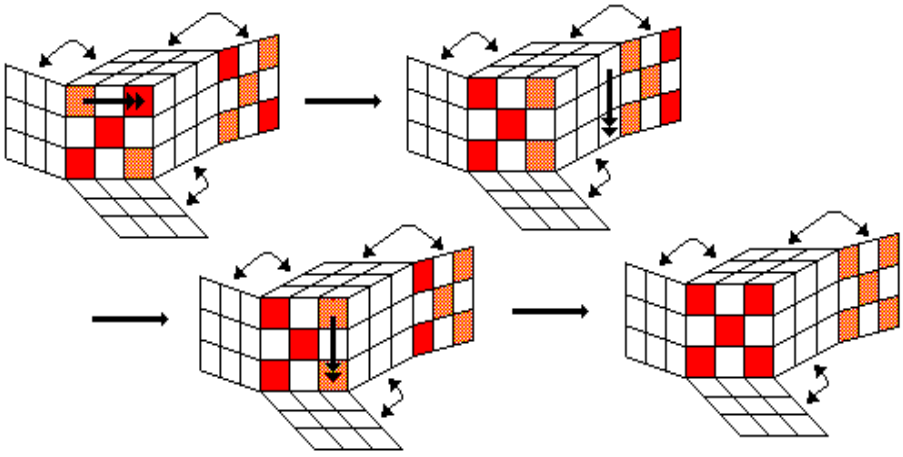


Figure 29

The next task is to get the red edge faces correct. If you alien the missing red edge face to be directly opposite to an orange edge face on the red side of the cube, then this face can be restored by using the Trick #12, as shown in Figure 30. Note that there are two possible ways to do this, as shown. This freedom may be important later. You should now have all of the red faces on the same side of the cube. Note that the orange faces are automatically all placed on the opposite side of the cube.

The next step in the solution is to restore another pair of opposite coloured faces. We will consider the blue and white faces as an illustration of what

may be required. Look at the face of the cube with the blue centre face. If none of the blue corner faces are on this side then double rotate both the red and orange faces as in Figure 31. If there are two blue corner faces correct and the situation is as in Figure 32 then you can recover them very simply as shown. The other possible situations and their solution are depicted in Figure 33.

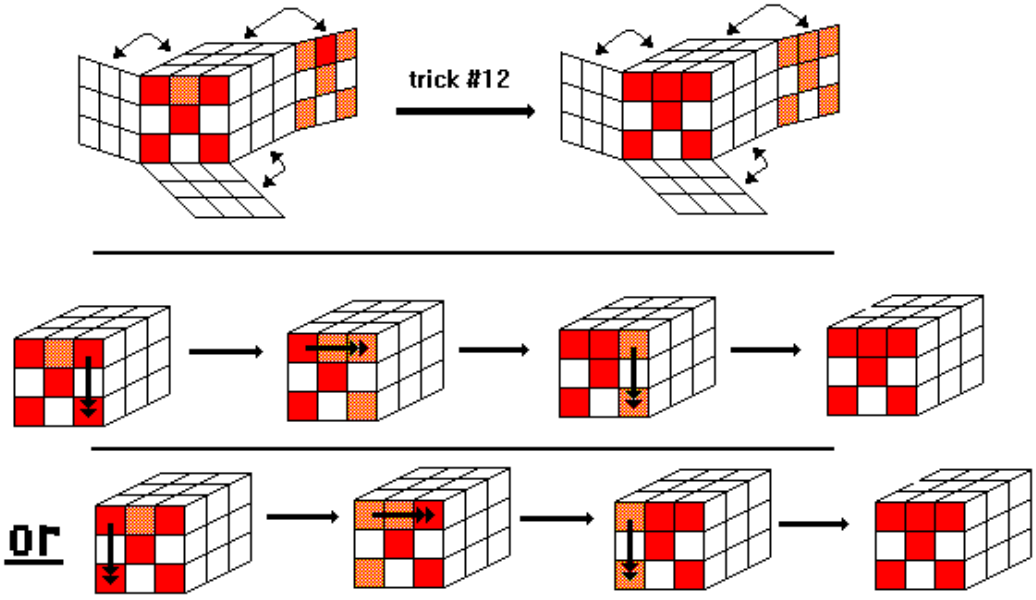


Figure 30

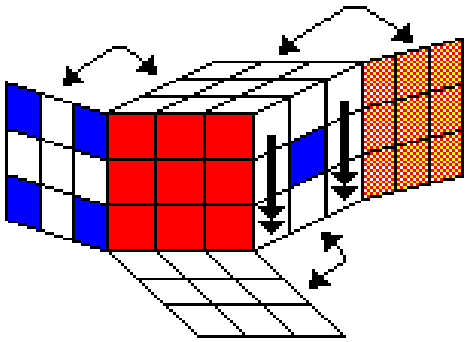


Figure 31

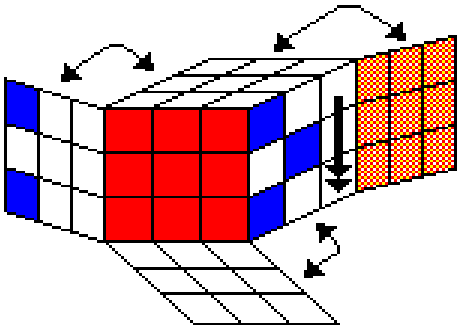


Figure 32

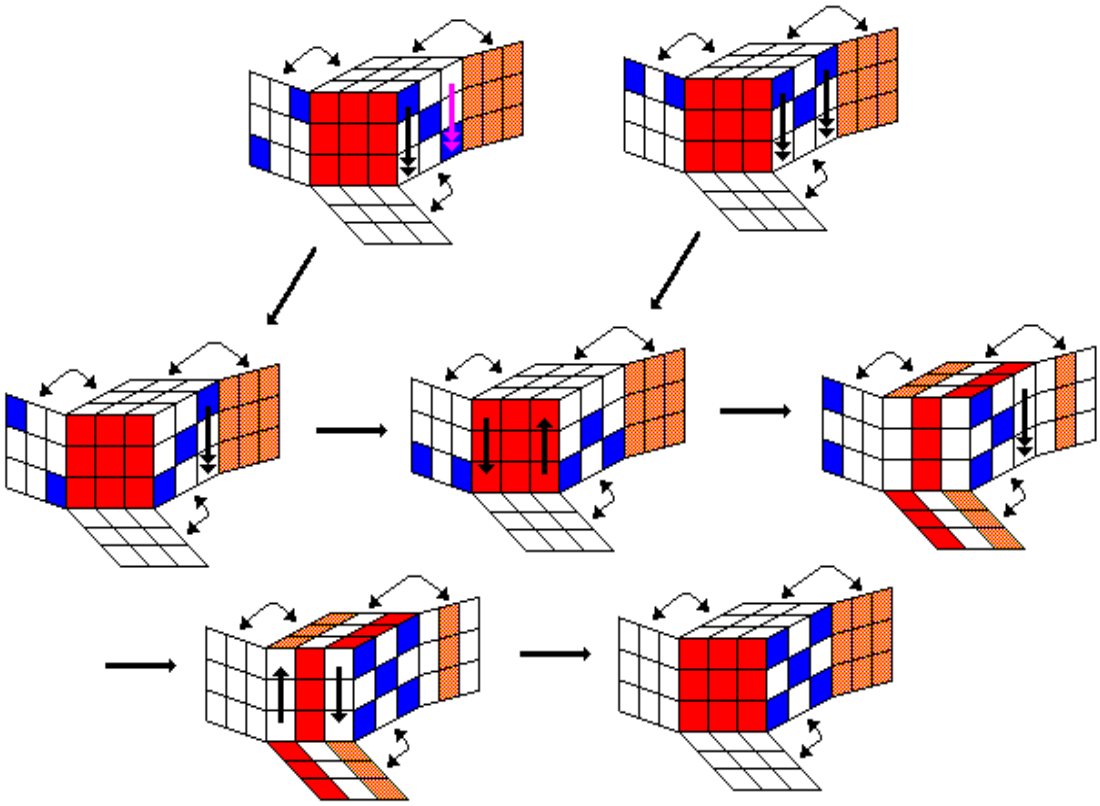


Figure 33

You should now be able to restore all of the blue edge faces that have a missing blue edge face directly opposite by using the same trick (#12) as was used to get the red edge faces correct. [Do not attempt to restore those blue edge faces that do not have a blue face directly opposite.] This process may mix the red and orange faces, but with columns only, and these faces can easily be restored as shown in Figure 34. Note that you can use this trick to restore the red and orange faces even if one of the blue edge faces is still not correct. Occasionally you may be left with one blue edge face to restore whose missing edge is not directly opposite, as shown in Figure 35. The solution to this case is depicted in Figure 35. You should now have four sides correct on the cube with the green and yellow corners also in correct position.

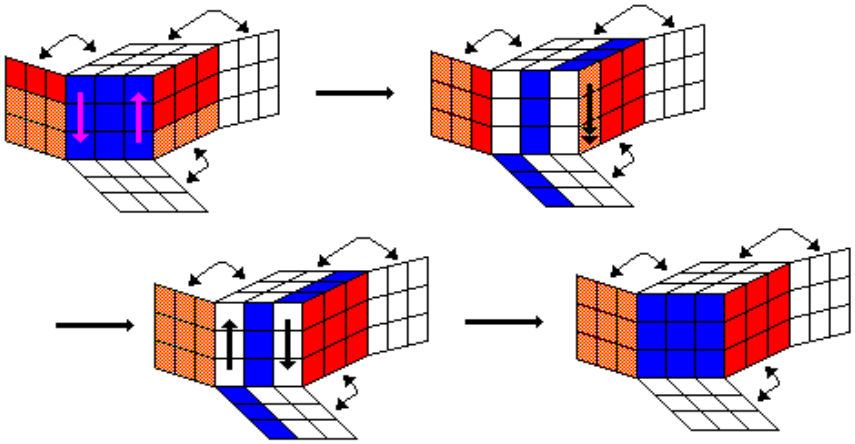


Figure 34

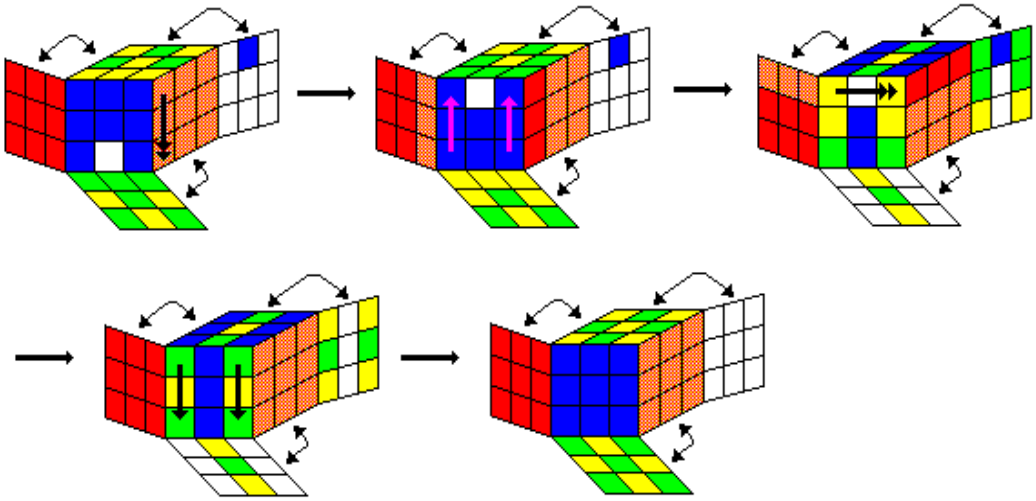


Figure 35

The only faces that may not be correct now will be some green and yellow edge faces. These can by restored by the same trick (#12) that was used to restore the red edge faces. In most cases you will find that you need to restore two such edge faces. A very typical situation is shown in Figure 36. Note that there are two ways to restore each edge face (see Figure 30) depending on which side of the incorrect edge face you first rotate. If you make the 'wrong' choice then you will finish up with columns on the other red, orange, blue and white faces. This can be corrected by reversing the moves that restore that edge face and then completing that edge face by rotating on the other side of that face.

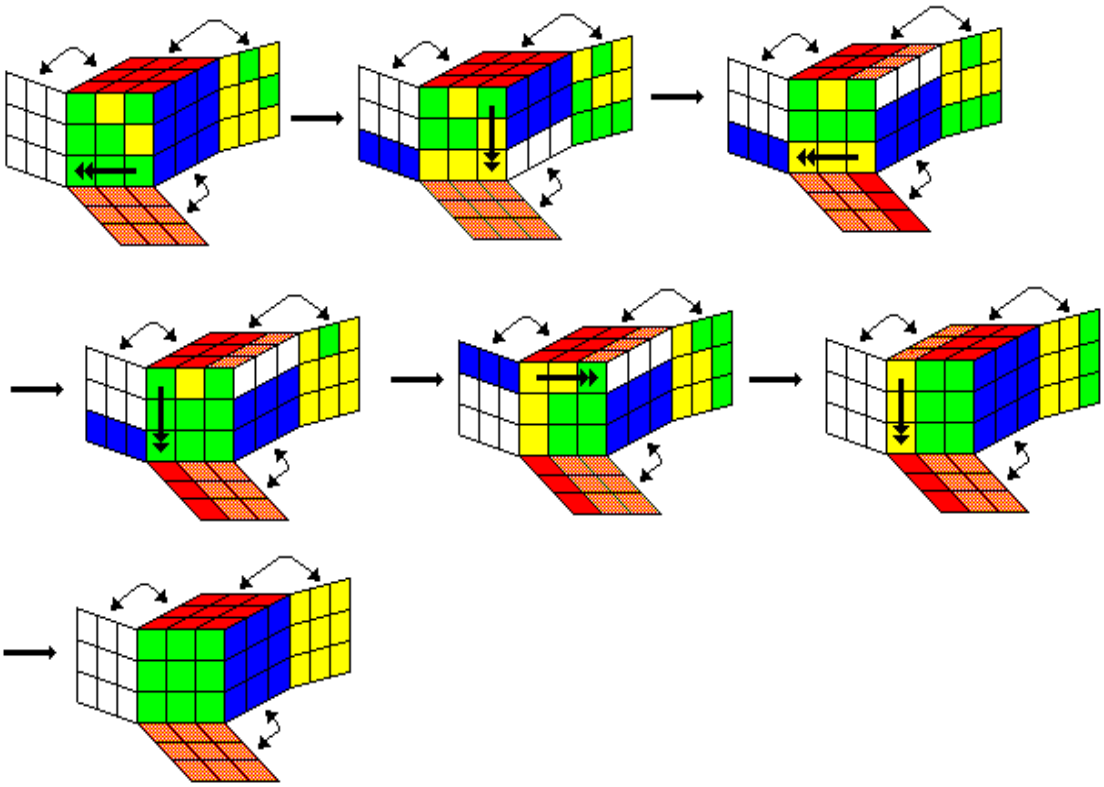


Figure 36

The cube should now be restored to the unscrambled state.

Other comments

Note that you do not need to do the red and orange faces initially. If, for example, the blue and white corner faces of the cube are almost correct then it may be much easier to do these faces first, followed by whichever other pair of faces that is closest to the unscrambled solution.

There are many other tricks and variations to the tricks shown in this book, which in special cases can accelerate the solution. We have tried, however, not to exhaust all such possibilities, but to give you sufficient information as to how to solve the cube puzzle. With a little familiarity and practice of the tricks shown in this book you should be able to complete the Rubik's cube puzzle in about 1-2 minutes without much difficulty.

One also anticipates that a much quicker solution may exist along the lines developed here, but where one gets the orientation of the corner pieces correct from the beginning. This would eliminate the possibility of twists. A detailed study of algorithms that manipulate corners may aid this endeavour. We encourage the interested reader to pursue this further.

Good luck with the cube.